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Matching Birthdays

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An interesting and well-known riddle in the area of probability is “The Birthday Problem.” You can find information about this problem in many introductory books in probability, probability and statistics, and math tricks books for laymen. This problem falls in the category of problems called counter-intuitive -- meaning that you may be surprised at the answer.

You can gather data and participate easily in any of your classes or group activities. Here’s **the problem: In a group with people of any age, what are the chances that two people in the group have the same birthday (disregarding the year of birth)?**

There are 365 days in a year and we won’t consider the complication of leap year – it wouldn’t make that much difference. It seems likely that the number of people in the room would have to be substantial to have a promising likelihood of two people matching the month and day of birth, wouldn’t you think?

Calculating the probability of at least two of n people having matching birthdays provides a good calculator exercise for students and an interesting challenge for beginning computer programmers.

Assumptions:

1. It is not leap year.
2. There are no twins in the room.
3. Disregard the year of birth.

Definitions:

n = the number of people in a room

P(2) = the probability that at least two people have matching birthdays

P(D) = the probability of no matches

Theory: Using this terminology, probability theory states

$$P(2) = 1 - P(D).$$

Also, if an event is made up of several sub-events, then the probabilities are multiplicative. That is, the probability of the event D can be found by multiplying the sub-events(a,b,c,...) leading to event D

$$P(D) = P(a) \cdot P(b) \cdot P(c) \cdot \dots$$

In order for event D to occur, sub-event a must occur AND sub-event b must occur AND sub-event c must occur AND so forth. Multiplying the probability of the sub-event will give us the

probability of event Z occurring. The probability of the each sub-event is less than one. Each time you multiply numbers less than one, the result is even smaller yet.

Procedure: Let's calculate P(D).

P(D) = the number of ways that nobody's birthday matches, divided by the number of ways that birthdays can be given to everyone in the room.

The denominator of P(D) is 365^n , since each person was born on one of the 365 days in the year. In the numerator, we must consider how many ways birthdays can be distributed to n people so that there are no matches. The first person can have any one of the 365 days for a birthday without restriction (with 365 ways out of 365 days). The second person can only have a birthday on one of the 364 days left (364 ways out of 365 days), and the third person must be born on one of the remaining 363 days (363 ways out of 365 days), and so forth.

Calculate the chances of two people having the same birthday:

This means for two people, the probability that they have different birthdays is

$$P(D) = \frac{(365)(364)}{(365)(365)} = \frac{(365)(364)}{365^2} = 0.9973$$

and: $P(2) = 1 - 0.9973 = 0.0027$

Therefore, with only two people (who are not twins) in the room, there is a 0.27% chance that they have the same birthday (that is, of course, if they are not twins!) Well, we knew that given only two people, it was highly unlikely that they would have the same birthday, didn't we?

In a room containing three people, calculate the chance that at least two of them have the same birthday:

For three people, the calculation looks like this

$$P(D) = \frac{(365)(364)(363)}{(365)(365)(365)} = \frac{(365)(364)(363)}{365^3} = 0.9918$$

and: $P(2) = 1 - 0.9918 = 0.0082$

With three people in the room, there is a 0.82% chance that two of them have the same birthday. Not much better really. But what happens as the number of people in the room grows? Just continue in the same fashion for more people.

In a room containing n people, calculate the chance that at least two of them have the same birthday:

For n people in the room,

$$P(D) = \frac{(365)(364)(363)\dots(365 - (n-1))}{365^n}$$

and: $P(2) = 1 - P(D)$.

Activities:

1. Given 365 possibilities for each person, what is the probability that at least 2 out of the n people match? Use a calculator or write a computer program to confirm the following:

N	Probability of at least one match	N	Probability of at least one match
2	0.0027	27	0.6269
3	0.0082	28	0.6545
4	0.0164	29	0.6810
5	0.0271	30	0.7063
6	0.0405	31	0.7305
7	0.0562	32	0.7533
8	0.0743	33	0.7750
9	0.0946	34	0.7953
10	0.1169	35	0.8144
11	0.1411	36	0.8322
12	0.1670	37	0.8487
13	0.1944	38	0.8641
14	0.2231	39	0.8782
15	0.2529	40	0.8912
16	0.2836	41	0.9032
17	0.3150	42	0.9140
18	0.3469	43	0.9239
19	0.3791	44	0.9329
20	0.4110	45	0.9410
21	0.4437	46	0.9483
22	0.4757	47	0.9548
23	0.5073	48	0.9606
24	0.5383	49	0.9658
25	0.5687	50	0.9704
26	0.5982		

2. Test this theory in each of your classes and clubs at school and in the community. Record the number of people in the room and the results of the birthday poll. Did any of the groups buck the odds?

Questions

1. What is the minimum number of people needed to have at least a 50/50 chance of at least one match? A 75% chance? A 90% chance?
2. Are there other circumstances where predictions can be made from a surprisingly small sample of people? (What about the media predictions from polls taken on election day?)
3. How many people would be needed in the group to be CERTAIN that two people will have the same birthday? Compare this with the number of people which gives a probability of 90%. Interesting, don't you think?

For more information:

Mosteller, Frederick. *Fifty Challenging Problems in Probability*. Dover pubs, ISBN 0-486-65355-2)

Paulos, John. *Innumeracy: Mathematical Illiteracy and Its Consequences*. pp 35-37.

The Birthday Problem, <http://silver.sdsmt.edu/~rwjohnso/module0.htm>